Cognitive resource allocation determines the organization of personal networks

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The typical human personal social network contains about 150 relationships including kin, friends, and acquaintances, organized into a set of hierarchically inclusive layers of increasing size but decreasing emotional intensity. Data from a number of different sources reveal that these inclusive layers exhibit a constant scaling ratio of $\sim$3. While the overall size of the networks has been connected to our cognitive capacity, no mechanism explaining why the networks present a layered structure with a consistent scaling has been proposed. Here we show that the existence of a heterogeneous cost to relationships (in terms of time or cognitive investment), together with a limitation in the total capacity an individual has to invest in them, can naturally explain the existence of layers and, when the cost function is linear, explain the scaling between them. We develop a one-parameter Bayesian model that fits the empirical data remarkably well. In addition, the model predicts the existence of a contrasting regime in the case of small communities, such that the layers have an inverted structure (increasing size with increasing emotional intensity). We test the model with five communities and provide clear evidence of the existence of the two predicted regimes. Our model explains, based on first principles, the emergence of structure in the organization of personal networks and allows us to predict a rare phenomenon whose existence we confirm empirically.

quantitative sociology | personal networks | complex systems

The analysis and modeling of social networks are a widely studied topic. A number of models have been proposed across different disciplines such as statistical physics and computer science (1), economics (2–4), statistics (5), or sociology (6, 7). All these models aim to explain commonly observed properties of social networks, including community structure, high clustering, degree correlations, etc. To this end, the focus is set on the macroscopic properties of the networks while keeping as simple as possible the assumptions made about the individuals involved. The macroscopic observables emerge then as a consequence of the interactions among the constituents of the system.

However, some of the most robust findings about human social networks go beyond these macroscopic quantities, being concerned with the size and organization of the individuals' personal networks. These studies suggest that, among humans, an individual typically deals with about 150 relationships including kin and friends (8–12). These relationships are further organized into a set of hierarchically inclusive layers of increasing size with decreasing emotional intensity (8, 12–16) whose sizes follow a characteristic sequence with a scaling ratio close to 3 (15): 5, 15, 50, 150. Although the overall size of the networks has been connected to our cognitive capacity (17), no theoretical explanation has been given for the layered structure and the consistent scaling ratio even though there is considerable evidence for their existence from many different media sources (18, 19).

In what follows we develop a mathematical model of the building blocks of social systems (social atoms) that should serve to connect the individual and collective perspectives of human societies. Indeed, whatever the (global) social structure is, it must comply with the (local) organization of the ego networks—just as any physical object must be consistent with its atomic composition. Our work therefore contributes to the literature and development of social physics—started in the 19th century by Comte and Quetelet—along the lines proposed in refs. 20 and 21 and other works aiming to study quantitative theories that yield testable predictions (see, e.g., ref. 22 for a detailed summary of the field before physicists themselves started to contribute to it). As we show below, our model not only accounts for the layered structure previously mentioned, but also predicts the existence of a contrasting regime. Indeed, we will see that depending on the relation between cost and available relationships, an inverted structure may arise in which layers with larger emotional content are also larger in size.

Model Description

In a population of N individuals, relationships (links) can be established out of a set of r different categories (that we later refer to as "layers") according to the strength of the links. This is the problem that we want to model, but in its bare bones, this is simply that of distributing a certain number of balls (links) in urns (layers)—in effect, a multinomial distribution. Of itself, this distribution yields no structure whatsoever but it is a reasonable

Significance

The way we organize our social relationships is key to understanding the structure of our society. We propose a quantitative theory to tackle this issue, assuming that our capacity to maintain relationships is limited and that different types of relationships require different investments. The theory accounts for well-documented empirical evidence on personal networks, such that connections are typically arranged in layers of increasing size and decreasing emotional content. More interestingly, it predicts that when the number of available relationships is small, this structure is inverted, having more close relationships than acquaintances. We provide evidence of the existence of both regimes in real communities and analyze the consequences of these findings in our understanding of social groups.

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Data deposition: The code and the data used in this paper have been deposited at https://github.com/1gnaci0/Cognitive.

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prior to assume as the default. In this setting, the probability that there are ℓ_k balls in urn $k \in 1, 2, \ldots, r$ will be

$$P_0(\boldsymbol{\ell}|N) = \frac{(N-1)!(r+1)^{-N+1}}{\ell_1!\ell_2!\cdots\ell_r!(N-1-\ell_1-\ell_2-\cdots-\ell_r)!}, \qquad [1]$$

where $\boldsymbol{\ell} = (\ell_1, \ell_2, \ldots, \ell_r)$.

Let us now assume that there is a cost s_k associated to each ball placed in urn k. Urns are initially all alike, so without loss of generality we can sort them by decreasing costs, $s_1 > s_2 > \cdots > s_r$. We now look for a probability distribution that is constrained to have a fixed average number of balls L and a fixed average amount of resources S to afford its costs; that is,

$$\sum_{k=1}^{r} \mathbb{E}(\ell_k) = \mathcal{L}, \qquad \sum_{k=1}^{r} s_k \mathbb{E}(\ell_k) = \mathcal{S}. \qquad [2]$$

To add this information to our prior, the procedure to follow is the maximum entropy principle (23, 24), as it is the only way to guarantee a posterior distribution that is compatible with the prior, compatible with the additional information, and unbiased (23, 25). The result is a distribution that measures the likelihood of different allocations of balls to urns with different costs

$$P(\boldsymbol{\ell}|\mathcal{S}, \mathcal{L}, N) = \mathscr{B}(L, \mathcal{L}/N, N) \binom{L}{\boldsymbol{\ell}} \frac{e^{-\mu \sum_k s_k \ell_k}}{\left(\sum_k e^{-\mu s_k}\right)^L}, \qquad [3]$$

with $\mathscr{B}(L, p, N) = \binom{N}{L} p^L (1-p)^{N-L}$ the binomial distribution and where μ, the only parameter of the model, arises as the Lagrange multiplier associated to the constraint on the total resources in Eq. **2** (*SI Appendix, Section 1C*).

Application to Ego Networks. Note that the model presented is fully general and applies to any situation in which a certain number of items of any sort have to be assigned to some categories with different costs. However, its connection with the organization of links within ego networks is rather natural. Although relationships change over time [they strengthen or weaken, new ones are created, and some old ones fade (26)], each individual handles a certain average number of links L at any one time (13). These relationships are further organized into different layers (urns, ℓ_k) according to the emotional strength (or closeness) of the links (for example, refs. 18 and 27 and references therein). Additionally, studies of both offline and online social networks indicate that time invested in interacting with individual alters seems to determine the emotional strength of the relationship (the higher the investment, the closer the relationship) (13, 28) and is thus largely responsible for their layered structure (8, 13, 14, 19, 27, 29, 30). These investments reflect the costs, s_k, that individuals have to make to create functional relationships. If we further assume a limited (cognitive) capacity S of individuals to handle relationships (8, 17), we have a problem to which the previous model applies. In what follows, we analyze which kind of testable predictions can be inferred from the organization of links in an ego network implied by Eq. **3**.

Results. We explore the emergence of structure, calculating the expected number of links in each layer. The ratio of this quantity between consecutive layers is

$$\frac{\mathbb{E}(\ell_{k+1})}{\mathbb{E}(\ell_k)} = e^{\mu|\Delta s_k|}, \qquad [4]$$

where $|\Delta s_k| = |s_{k+1} - s_k| > 0$ is the cost difference between them.

Eq. **4** identifies two distinct regimes according to whether $\mu > 0$ or $\mu < 0$:

- If $\mu > 0$, then $\mathbb{E}(\ell_{k+1}) > \mathbb{E}(\ell_k)$, and the most expensive layers will be less populated than the less expensive ones. We call this the standard regime.
- If $\mu < 0$, then $\mathbb{E}(\ell_{k+1}) < \mathbb{E}(\ell_k)$, and the most expensive layer will be the most populated one. We call this the inverse regime.

Let us now consider that $|\Delta s_k|$ is a constant so that costs s_k decrease linearly with k. We can then write $s_k = s_1 - (s_1 - s_r)(k-1)/(r-1)$, where r is the number of layers and $s_1 > s_r > 0$. In this scenario, the value of μ is determined by the value of S/L according to (see *SI Appendix, Section 1D*, for details)

$$\frac{s_1 - \mathcal{S}/\mathcal{L}}{s_1 - s_r} = f(\hat{\mu}) \equiv e^{\hat{\mu}} \frac{(r-1)e^{r\hat{\mu}} - re^{(r-1)\hat{\mu}} + 1}{(r-1)(e^{r\hat{\mu}} - 1)(e^{\hat{\mu}} - 1)}, \qquad [5]$$

where we define $\hat{\mu} \equiv \mu(s_1 - s_r)/(r-1)$ for convenience. Note that the choice $|\Delta s_k| = 1$ implies that $\hat{\mu} = \mu$, so we use both interchangeably.

Hence, which regime an individual belongs to depends on the ratio S/L (Fig. 1), and this in turn depends on the total number of social relationships that an individual has. If L is large, this structure will be standard. This is what has been

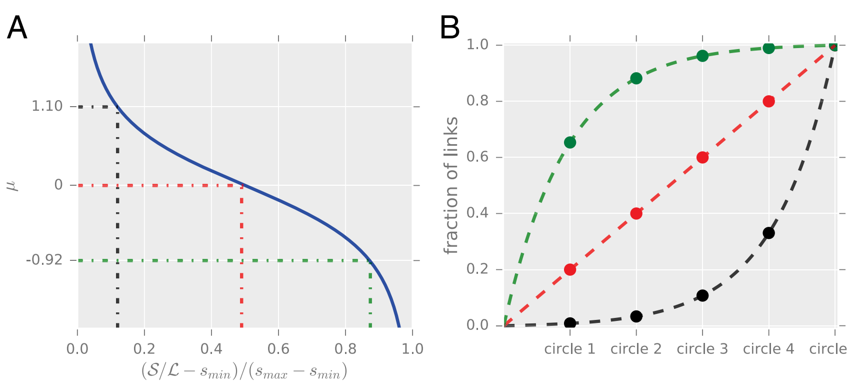

Fig. 1. The two regimes as a function of the mean cognitive cost allocatable per link. (*A*) Dependency of the parameter μ with the ratio S/L. The blue line represents the typical dependency of the parameter with the mean cognitive cost S/L that an individual can spend in maintaining a link. As a reference, it has been computed with Eq. **5** for $r = 5$ circles and $\Delta s_k = 1$, but it is representative of the expected behavior. Given a fixed cognitive capacity S, increasing L implies moving to the left in the graph. The particular value of S/L determines the value of μ. Dotted lines represent example cases (fixed S); in green, an individual with "few" alters (inverse regime; $\mu = -0.92$); in red, the limit case (change of regime; $\mu = 0$); and in black, an individual with "many" alters (standard regime; $\mu = 1.10$). (*B*) Expected regimes as a function of μ. The colors follow the specifications given in *A*. That is, the black dashed line represents the standard regime, the red one the limit case, and the green one the inverse regime. Solid circles represent the expected fraction of links in each circle for the different examples.

observed in most studies analyzing the organization of ego networks (8, 14), and it is what seems reasonable to expect: The less costly a relationship is, the more of them one can have. If, on the contrary, L is small, the structure is inverted. In other words, the more time or cognitive capacity that an individual has, the more he/she is able to devote to strengthening all his/her relationships.

Following the customary use in the anthropological literature, we define circle k as including all links from layers $1, 2, \ldots, k$. Thus, we use the term circle as a proxy for proximity, so that egos have closer (more costly) relationships with alters in the inner circles than with those in the outer circles. The fraction of links in circle k is then given by

$$\chi_k = \frac{1 - e^{k\mu}}{1 - e^{r\mu}}. \qquad \textbf{[6]}$$

In the standard regime ($\mu > 0$) $\chi_k \approx e^{(k-r)\mu}$, and we recover the constant scaling ratio $\chi_{k+1}/\chi_k \approx e^{\mu}$ between consecutive circles that has been extensively reported in the literature (8, 14). However, in the inverse regime ($\mu < 0$) χ_k quickly approaches 1 as k increases, implying that most links are within the innermost circle. Therefore, a second, as yet unnoticed, regime is predicted in which the structure is reversed—the more demanding the layers, the more populated they are.

This regime is expected to arise when the ratio S/L is particularly large. That would be the case, for instance, of individuals living in small populations or within limited social environments. Assuming that the capacity of those individuals is similar to the capacity of individuals elsewhere, the reduced number of possible relationships should be translated into an inversion of their circles, making apparent the inverse regime predicted by the model. This phenomenon can in fact be observed in data collected during an oceanic scientific expedition (16) and, perhaps, within a community of immigrants (31)—but it seems that neither of these studies were aware of this.

Empirical Validation

We test our model on five communities. One is a community of college students (where we expect the standard regime to predominate) and the other four are communities of immigrants (where we expect the inverse regime to be more common because they are likely to lack opportunities to make friendships outside their respective communities). Note that we use the term community here in a broad sense, as a group of people living in the same place or having a particular characteristic in common. Importantly, our model is defined at the individual level, and the background information (such as the community an individual belongs to) is merely used to conjecture what regime should prevail in each case.

Standard Regime. We begin by analyzing data from a group of ($n = 84$) students from a major Middle Eastern university (32). In this community we anticipate that most individuals will follow the standard regime, as in previous studies (14, 15). During the experiment, every participant had to classify his/her relationships with everyone else using a 0–5 scale. We use this information as a proxy of the relative cost of each relationship and build the personal networks accordingly (*Materials and Methods*).

The results are summarized in Fig. 2. Most individuals ($\sim$98%) have a value of $\mu > 0$, meaning that their circles show the standard structure (Fig. 2*B*), as expected. These values of μ (Fig. 2*A*) are grouped around a central value $\mu = 0.978$, corresponding to a scaling ratio of $x \equiv e^{\mu} = 2.66$ (3.13 if we average the xs instead), in agreement with previous studies (8, 14, 15, 19). However, the data also allow us to detect a small proportion ($\sim$2%) of individuals whose networks lie within the inverse regime (Fig. 2*C*).

Inverse Regime. To elicit the inverse regime we focus on four different communities of immigrants (see *Materials and Methods* and *SI Appendix, Section 2*, for details). The first one derives from a sociological study conducted in 2008 in Roses (33), a small town of about 20,000 inhabitants in Girona (Catalonia, Spain). This study sampled ($n = 25$) personal networks within a community of approximately 80 Bulgarian immigrants. The remaining three derive from a different study carried out in Barcelona (Catalonia, Spain) (34, 35). In this case, the focal groups were communities of Chinese (sampled $n = 21$), Sikh ($n = 24$), and Filipino immigrants ($n = 25$), each numbering some thousands of individuals.

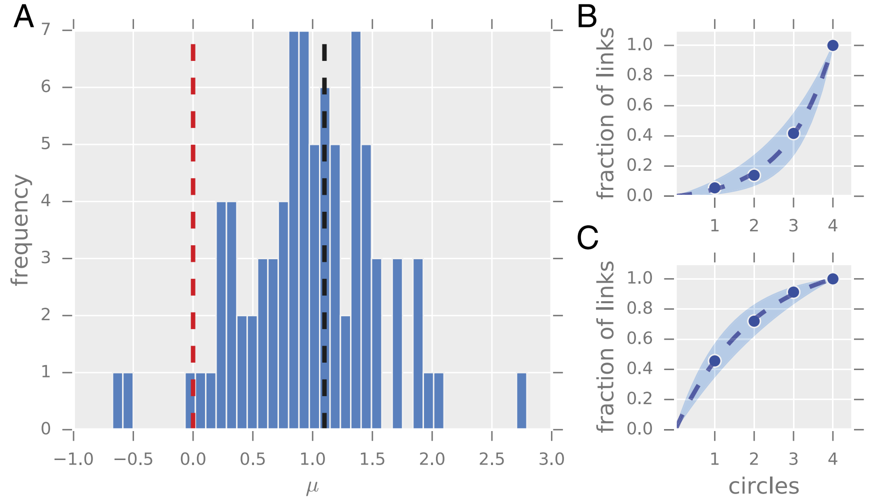

Fig. 2. Summary of the results for the community of students. *B* and *C* show representative fittings for each of the regimes. Solid circles represent experimental data, blue dashed lines represent the graph of Eq. **6** with the corresponding estimated parameter, and shaded regions show the 95% confidence interval for that estimate (*Materials and Methods*). *A* shows the distribution of the parameter estimates, μ. In the analysis of the community of students we did not take into account the scores 0 and 1. We also excluded one individual who had no alters in the considered layers (*Materials and Methods*). (*A*) Distribution of the parameter estimates for the community of students ($n = 83$). The black, dashed line indicates the typically observed scaling ratio $e^{\mu} = 3$ ($\mu = 1.099$). The red, dashed line marks the change of regime $\mu = 0$. (*B*) Representative fitting for an individual exhibiting the standard regime, with layers $\ell = (2, 3, 10, 21)$ and estimated parameter $\mu = 0.846$. (*C*) Representative fitting for an individual exhibiting the inverse regime, with layers $\ell = (26, 15, 11, 5)$ and estimated parameter $\mu = -0.503$. A comprehensive set of figures, including fittings for every subject, is available in *SI Appendix, Section 3*, Figs. S1–S6.

In all studies, each participant nominated 30 alters using a free name generator, alongside his or her self-rated perceived social (or emotional) proximity to each individual alter on a 1–5 scale (*Materials and Methods*). This kind of generator tends to elicit strong links at the beginning, but the list is long enough to gather information from other types of relationships, including weak links (35). Thus, this methodology would be able to capture either the standard or the inverse regimes.

Although there are differences among the sampled groups, all of them belong to well-differentiated communities within their respective social environments, which tend to use and preserve their native languages and traditions and form a support network for their members. Indeed, one of the main mechanisms for the formation of these communities is that the individuals already settled in the hosting location serve as links for those yet to come. This process facilitates the integration of the newcomers in the host country in terms of professional and housing opportunities, among others (see refs. 33–35 and *SI Appendix, Section 2*, for details). All these facts suggest that these communities form independent, small-scale social environments within their places of residence—hence perfect candidates for the context in which an inverse regime might hold.

Fig. 3 summarizes our results for these communities. Remarkably, 75% of the networks analyzed lie within the inverse regime with $\mu < 0$, confirming our hypothesis. Furthermore, the remaining 25% present values of μ very close to 0, significantly lower than in the case of the community of students (compare Figs. 2 and 3).

The case of the Bulgarians (Fig. 3 *A* and *a*) is particularly striking since this percentage goes up to 96% of the networks. As noted before, this sample was taken from a community of only about 80 individuals, a small population by any standards. Indeed, the researchers of the original study (33) concluded that the context of Roses allowed the community to form a denser and more homogeneous ethnic network than in larger towns (i.e., Barcelona).

The Sikhs (Fig. 3 *B* and *b*) and the Chinese (Fig. 3 *C* and *c*) show similar percentages, 88% and 86%, respectively, whereas for the Filipino community (Fig. 3 *D* and *d*) this number is significantly lower (68%). These differences might be a consequence of a number of sociological and cultural factors that we do not discuss here (see *SI Appendix*, section 2 for a brief description and refs. 33–35 for details). In either case, our results suggest that the subjective number of available contacts is in fact smaller than average in all of the communities, resulting in a predominance of inverted personal networks. It is important to stress that the type of structure that was predicted, hierarchical inclusive layers of increasing intensity with increasing size, had been so far only anecdotally suggested in the literature (16, 31).

Discussion

We present a simple model of social interaction that naturally reproduces the layered structure of personal social networks in which each successive layer includes disproportionately more alters. This model reveals an unexpected finding, namely that in a proportion of cases an inverse structure emerged in which more alters are found in the inner layers. These inverse structure networks are associated with smaller than usual networks and seem to imply that individuals have a (more or less) fixed quantity of cognitive capital (indexed as time available for investing in alters) which they can spread either thinly among many alters or thickly among fewer alters. Which option an individual opts for may depend on his or her personality [introverts opting for smaller networks (36)] or on the size of the community he or she happens to be a member of (as seems to be the case here with the immigrant communities where opportunities for social

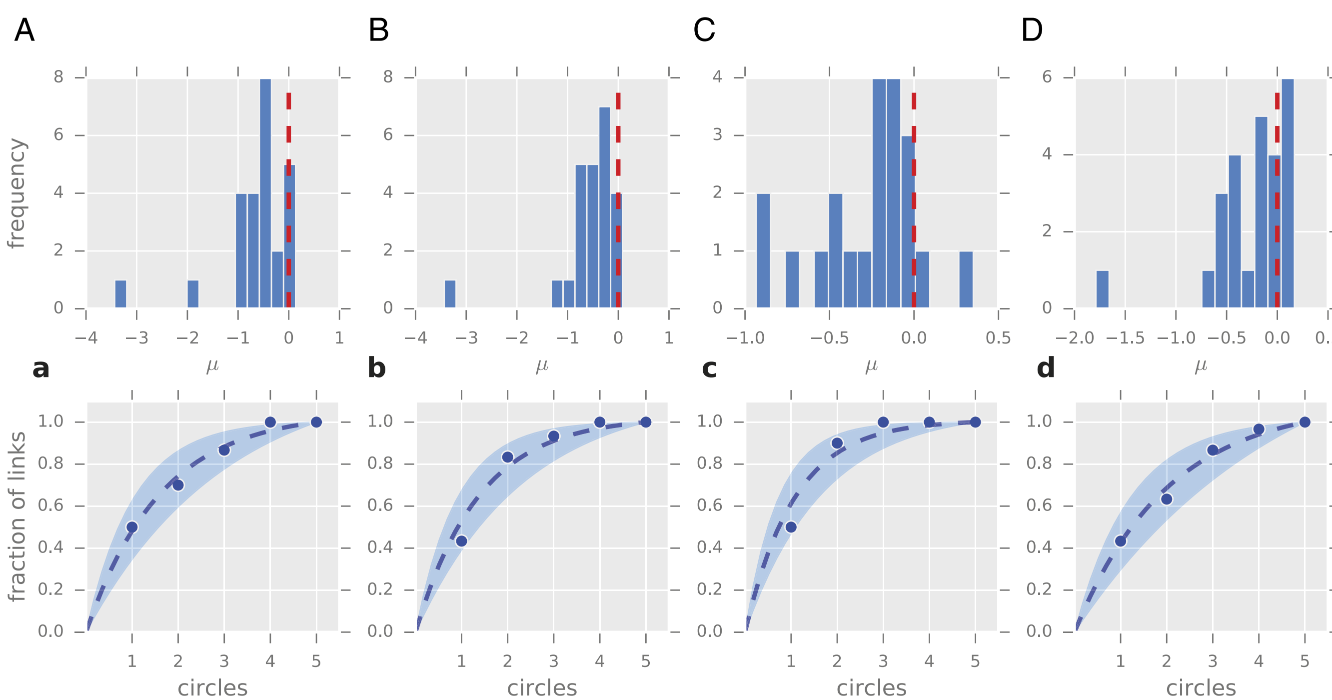

Fig. 3. Summary of the results for the communities of immigrants. *A–D* (*Upper*) show the distributions of the parameter estimates for the communities of immigrants. The red, dashed lines mark the change of regime (i.e., $\mu = 0$). *a–d* (*Lower*) show examples of fittings for individuals in each community. Solid circles represent experimental data, blue dashed lines represent the graph of Eq. **6** with the corresponding estimated parameter, and shaded regions show the 95% confidence interval for that estimate (*Materials and Methods*). (*A*) Distribution of the parameter estimates for the community of Bulgarians ($n = 25$). (*a*) Example of fitting for an individual in the community of Bulgarians with layers $\ell = (15, 6, 5, 4, 0)$ and $\mu = -0.616$. (*B*) Distribution of the parameter estimates for the community of Sikhs ($n = 24$). (*b*) Example of fitting for an individual in the community of Sikhs with layers $\ell = (13, 12, 3, 2, 0)$ and $\mu = -0.727$. (*C*) Distribution of the parameter estimates for the community of Chinese ($n = 21$). (*c*) Example of fitting for an individual in the community of Chinese with layers $\ell = (15, 12, 3, 0, 0)$ and $\mu = -0.934$. (*D*) Distribution of the parameter estimates for the community of Filipinos ($n = 25$). (*d*) Example of fitting for an individual in the community of Filipinos with layers $\ell = (13, 6, 7, 3, 1)$ and $\mu = -0.496$.

interaction seem to be restricted to a small number of individuals by social exclusion).

Note that all distributions (Fig. 2*A* and Fig. 3 *A–D*) show a large dispersion, implying that the structure of circles is quite personal (and depends, among other things, on the individual's number of links). This confirms an earlier empirical finding suggesting that individuals allocate their social effort in quite different and consistent ways, such that each is characterized by a kind of "social fingerprint" (26). In fact, taking both results together, the parameter of our model may serve as a quantitative characterization of such a fingerprint.

Although we have presented these results in terms of layers or circles, a simple modification of the current model gets rid of the layers to classify ties in a continuum (*SI Appendix*, section 1F), thus reproducing what is typically seen in most personal social networks [i.e., individual alters can be listed in a continuous list of emotional closeness and/or contact frequency (9)]. The two structural regimes obtained when there are discrete layers also arise in this version. Thus, whether we view egocentric networks in terms of layers or as an ordered linear sequence of dyadic relationships simply reflects different (equally valid) ways of describing an individual's personal social network.

One possible criticism of these results is that they may be an artifact of the way the questions are posed in the surveys—people are usually asked to classify their relationships in predefined categories. However, a number of approaches have been taken and yield much the same pattern in different types of datasets: Online social networks such as those based on Twitter or Facebook as well as those based on phone calls (14, 19) yield exactly the same layered organization as we find in self-rated questionnaire-based ratings (8, 13, 15).

Another possible criticism could be that the data we used in our empirical validation were obtained using different methodologies (*Materials and Methods*). Nevertheless, there are examples in the literature that suggest that the influence of the different protocols is not significant. Studies with larger source populations (i.e., more choices available) (17, 37) have shown that, even when using an open-ended method, individuals list only about 10–30 people, and the structure found was still the standard regime. This is because imposing a cutoff on a standard network does not change it into an inverse structure: This is clear from Fig. 2*B* where a cutoff at, say, layer 2 or 3 would not change the form of the distribution into that shown in Fig. 2*C* (examples also given in ref. 26). Note that the name generator used with the groups of immigrants had a limit of 30 names (*Materials and Methods*), and we nonetheless found both standard and inverse regimes. Additionally, the data reported in the shipboard survey mentioned above (16), where individuals living in a boat were asked about their relationships with other members of the expedition (a protocol similar to that used with the community of students that we use in this paper), suggest average sizes of 14.6 and 26.7 individuals for the first and second circles, respectively, much as would be expected for an inverse regime. The inverse regime is precisely what we would have expected to emerge in this setting. Therefore, although further studies should investigate the impact of different protocols, there is no a priori reason to suppose that either methodology would bias the results in any particular direction.

It may be surprising that there are individuals whose ego networks show an organization opposite to what is typical in their contexts. There are several reasons why this might be so, all of which derive from the fact that the potential size of the ego network is constrained. As such, these are predictions of the model that could be tested. One is that an individual's cognitive capacity [the ability to manage many relationships, which is a function of an individual's brain size (17, 38, 39) or intellectual ability (40) or the time costs of investment in ties (28, 41)] is limited or because the available population is small (for geographical or, as in the case our immigrant samples, social reasons). Network size might also vary with personality differences. Introverts, for example, typically have significantly smaller egocentric social networks than extroverts (36). In such cases, introverts have smaller but emotionally more intense relationships on average than extroverts or those with large networks, who seem to spread their available cognitive capital more thinly (9, 36). This seems to be due to a constraint on available social time that applies across all individuals (42).

More interestingly, perhaps, our model predicts how the increasing availability of online social networks may affect the way we handle our relationships. Since these technologies reduce the effective cost of maintaining some relationships, it should be easier for individuals to establish larger networks and this should promote the standard regime. However, if online relationships are cheaper to maintain because they obviate the costly business of physically meeting up with an alter (41), it follows that any increase in online network size will be associated with a reduction in average tie strength. This would incentivize weak relationships, which might well be another reason why the inverse regime has remained largely unnoticed until now.

Finally, from a socio-centric perspective, our model suggests a way to identify whether an interconnected set of individuals (i.e., a community in the technical network analysis sense) is "small" or not, namely according to the regime of their ego networks. Consider as a reference a layered structure $\boldsymbol{\ell} = (5, 10, 35, 100)$ (giving the typical structure of circles: 5, 15, 50, 150) and an arbitrary linear decrease in the costs. In such a setting we find that the change of regime happens at a network size of 88 and that there is a maximum network size of 220 [a value close to the maximum observed network size of ~ 250 (8, 36)]. We also find that communities with sizes less than or equal to 55 members will have most of their contacts in the inner circle (thus, forming an absolutely cohesive group). This latter finding is of particular interest, because groupings of ~ 50 occur frequently in small-scale traditional societies: This is the typical size of hunter-gatherer bands (overnight camp groups), a grouping of special functional importance in terms of foraging and protection against predators (43). It also represents the primary functional social grouping in personal social networks, being the set of alters to whom an ego devotes most of his or her social time and effort (9, 13). More interestingly, perhaps, communities built up on a mixture of the two regimes might exhibit quite different properties from the socio-centric point of view. They might also gel less well and hence be less stable. Exploring these differences may shed light, for instance, on our understanding of the internal structure of human societies and the reasons why natural communities fission when they do (44).

Materials and Methods

Reciprocity Survey. In this survey (32), 84 students (60% female and 40% male) from a major Middle Eastern university volunteered to participate. Each participant was presented with a list of the other 83 participants and was asked about his/her relationship with each one of them. The question we are interested in was stated as follows: "How close are you to this person?". And the options were the following: "0, I do not know this person"; "1, I recognize this person but we never talked"; "2, acquaintance (we talk or hang out sometimes)"; "3, friend"; "4, close friend"; and "5, one of my best friends." For each participant we store the number of answers of each type in an array (ℓ_k), so that ℓ_{6-k} is the number of type k answers. These numbers are our representation of the layers. For the analysis presented in this paper we excluded the cases scored with either 0 (no relation whatsoever) or 1. The latter are excluded for two reasons: (*i*) Recognizing someone but having never talked with him or her hardly counts as a meaningful relationship, and (*ii*) there surely are other people outside this sample that the surveyed subjects recognize but never talked to, but are not part of the survey (limited to the 84 students) (see *SI Appendix*, *Section 3*, and Fig. S2, for a complete set of figures considering five instead of four layers).

Communities of Immigrants. The data were collected in a similar way between November 2008 and April 2009 in all four immigrant studies, using the open source software EgoNet. In the case of the Bulgarians, the following name generator was used (33): "Tell us about 30 people who you know on a first name basis, with whom you have had contact in at least the last two years and who you could contact again if necessary. It is important that all categories of contacts (family, friends, workmates [. . .]) be represented." For the remaining three communities, the name generator was (34, 35) the following: "Tell us 30 people you know by name, and vice versa. It can be everyone. Try to mention people important for you, but also other people not so close but whom you meet frequently. Try to use pseudonyms, but be sure you can recognize them later." In both cases each participant rated the perceived closeness of his/her relationship with each alter. The options were as follows: "1, not close at all"; "2, not very close"; "3, quite close"; "4, close"; and "5, very close." With this information we create an array as before.

Code Implementation and Data Availability. All numerical analysis is carried out in Python with the packages scipy.optimize and scipy.integrate (see the documentation for details). The code and the data used in this paper are available at https://github.com/1gnaci0/Cognitive. The original data from the Chinese, Sikh, and Filipino communities (34) are available at visone.info/wiki/index.php/Signos_(data). The data from the community of students (32) can be found at dx.doi.org/10.1371/journal.pone.0151588. The original data from the community of Bulgarians are not publicly available, but we provide an anonymized version with the information relevant to our work.

Estimate of the Parameter. The maximum-likelihood estimate for μ is obtained by numerically solving the equation $\frac{L_1}{L} = (r-1)f(\mu)$, where $L \equiv \sum_{k=0}^{r-1} \ell_{k+1}$, $L_1 \equiv \sum_{k=0}^{r-1} k\ell_{k+1}$, and $f(\mu)$ is given by Eq. **5** (see *SI Appendix, Section 1E*, for a full description of the above expressions). We used fsolve with tolerance 10^{-6} for the relative error between two consecutive iterates.

Limit cases. The model presents singularities when all of the relationships happen to be in either the first layer [then $f(\mu) = 0$, which holds for $\mu \to -\infty$] or the last layer [then $f(\mu) = 1$, which holds for $\mu \to +\infty$]. The data from the reciprocity survey include one individual (no. 80) with this sort of structure, so we excluded this datum from our analysis.

Confidence interval. To find the $1-\delta$ confidence interval we have to compute the cumulative distribution (see *SI Appendix, Section 1E*, for details) $G(t|\boldsymbol{\ell}) = \int_{-\infty}^{t} P(\mu|\boldsymbol{\ell})\,d\mu$, which involves integrating

$$F_t(R) \equiv \int_0^t \left(\frac{1-e^{-\mu}}{1-e^{-\mu r}}\right)^L e^{-\mu R}\,d\mu = \frac{1}{R}\int_0^{tR} \left(\frac{1-e^{-z/R}}{1-e^{-zr/R}}\right)^L e^{-z}\,dz. \quad [7]$$

For finite values of t we use quad. For $t \to \infty$ we evaluate the integral using a Gauss–Laguerre quadrature with 150 points. The extremes of the confidence interval $[t_1, t_2]$ are obtained by solving $G(t_1|\boldsymbol{\ell}) = \delta$ and $G(t_2|\boldsymbol{\ell}) = 1-\delta$. To that end, we use fsolve with tolerance 10^{-6}. The results presented in this paper consider $\delta = 0.025$ (95% confidence interval).

Numerical stability. Overflows in Eq. **7** due to the exponentials are avoided by evaluating the logarithm of the integrand. The singularity at $\mu = 0$ is avoided by Taylor expanding $e^{-\mu r}$ and $e^{-\mu r}$ up to third order. The singularity at $\mu = 0$ of Eq. **6** is avoided by using the Taylor expansion $\chi_k \approx k/r + (k/2r)(e^{\mu}-1)(k-r)$ for $|e^{\mu}-1| \le 10^{-6}$.

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